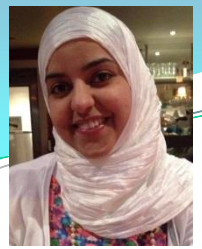

Mathematical Model of Brain Tumor: The Invasion of Glioma depend Blood Vessels

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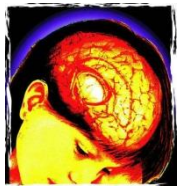
Introduction



Gliomas form the most dominant class of primary brain tumour, arising from the supporting glial cells that provide the majority of cells of the central nervous
system (CNS).

Regional variation in brain blood vessels density

The vascular supply to different brain regions is not homogeneous, and large differences in capillary density exist between grey and white matter, and among the brain lobes figure1. However, the interaction between the brain vascular and glioma invasion is poorly understood. The aim of this research was to investigate the effects of blood vessels density (BVD) on glioma invasion.

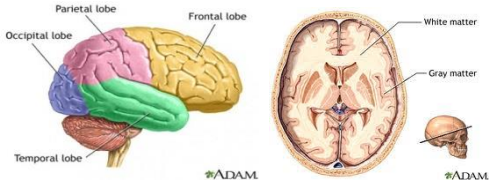


Figure 1: Lobes of the Brain (left), grey and white matter of brain (right). (From: http://www.md-health.com/Lobes-Of-The-Brain.html http://www.nlm.nih.gov/medlineplus/ency/imagepages/18117.htm)

Mathematical Model

- Tumour Cell, c, movement at microscopic level: the transition probabilities, τ, depend on local densities, $\bar{u}_n$.

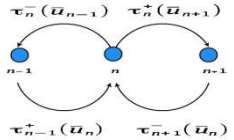


Figure 2: A diagram of the transition probabilities on 1D lattice.

Macroscopic level: reaction - diffusion type model

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}(\tau(v)c_x - c\tau_x(v)) + r(v)c(1-c)$$

$$\tau(v) = \frac{\alpha + \beta_1 v}{\gamma + \beta_2 v} \quad , \quad r(v) = rv$$

The model assume positive impact of BVD on Glioma invasion when $\beta_1 > \beta_2$ and negative impact vice Versa. Figure 3 (below) shows the analytical minimum wave speed vs BVD for different sizes of $\beta_{1,2}$

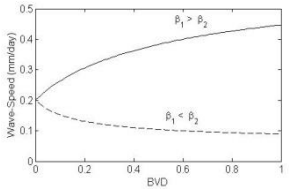


Figure 3: Tumour wave speed vs BVD with mid-proliferative rate

Simulation

- $\beta_1 > \beta_2$

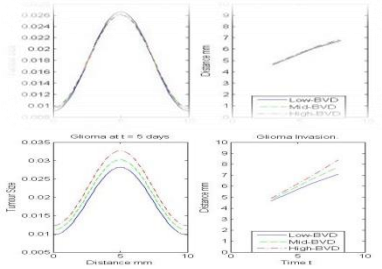


Figure 4: Glioma size (left) and invasion (right) with low top row) and high (bottom row) proliferation rate.

- $\beta_1 < \beta_2$

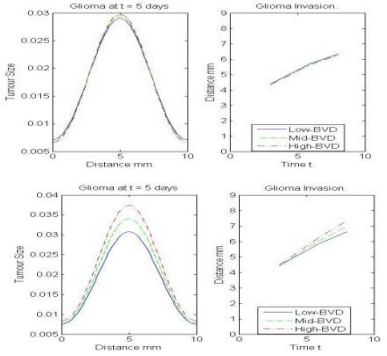


Figure 5: Glioma size (left) and invasion (right) with low (top row) and high (bottom row) proliferation rate.

Conclusions

- Our spatio temporal model take into account the heterogeneity of blood vessels as a factor of the glioma invasion.
- The model simulation run under zero flux boundary condition.
- Glioma with high proliferative rate has the big size and invasion.
- The model can be extend to consider the lack of oxygen (hypoxia).

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Acknowledgments

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